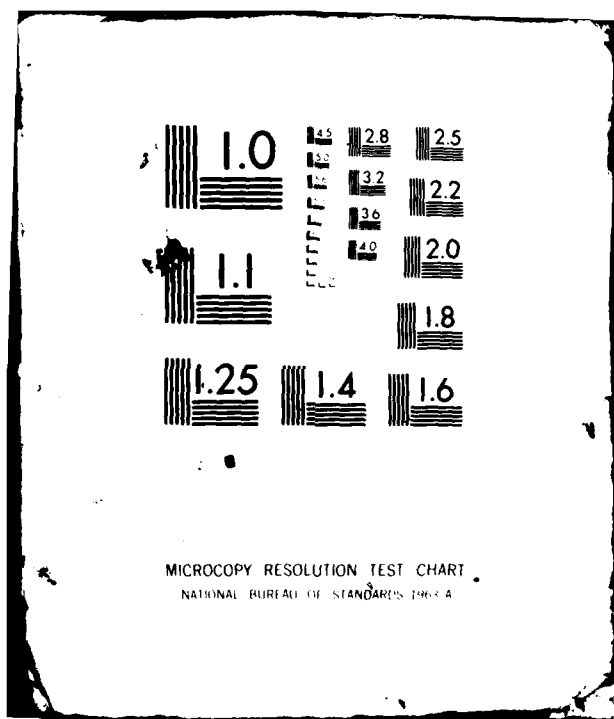


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PREDICTION OF DRIFT FORCES ON TWIN HULL BODIES IN WAVES.(U)

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## TABLE OF CONTENTS

|   | Page |
|---|------|
| LIST OF FIGURES . . . . .                             | 111  |
| LIST OF TABLES . . . . .                              | 111  |
| ABSTRACT . . . . .                                    | 1    |
| INTRODUCTION . . . . .                                | 1    |
| ANALYSIS OF MEAN FORCES AND MOMENTS . . . . .         | 2    |
| RESULTS AND DISCUSSIONS . . . . .                     | 6    |
| CONCLUSION . . . . .                                  | 14   |
| ACKNOWLEDGMENT . . . . .                              | 14   |
| APPENDIX I - DERIVATION OF MEAN ROLL MOMENT . . . . . | 15   |
| REFERENCES . . . . .                                  | 22   |

### LIST OF FIGURES

|  |    |
|--|----|
| 1 - Description of Coordinate System and Wave-Heading Angle ( $\mu$ ) . . . . .                            | 3  |
| 2 - Motion Amplitude for Beam, Bow Quartering and Head<br>Waves for SWATH . . . . .                        | 8  |
| 3 - Steady Mean Forces and Moments for Beam, Bow-Quartering and<br>Head Waves for SWATH . . . . .          | 10 |
| 4 - Steady Mean Sway Force and Roll Moment for Restrained and<br>Unrestrained Condition for SWATH. . . . . | 12 |
| 5 - Steady Mean Sway Force of SWATH and Monohull Ship in<br>Beam Waves . . . . .                           | 14 |

### LIST OF TABLES

|  |    |
|--|----|
| 1 - Particular Dimensions of SWATH and Monohull in Full Scale . . . . .                      | 7  |
| 2 - Maximum Steady Mean Forces and Moments for $\lambda/\zeta_A = 60$<br>for SWATH . . . . . | 13 |

PREDICTION OF DRIFT FORCES ON TWIN HULL  
BODIES IN WAVES

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A theoretical method is presented to predict the steady mean surge and sway forces and roll and yaw moments on floating twin-hull ships in waves. The steady mean forces and moments are derived by utilizing the conservation of momentum of the fluid surrounding the body. The associated velocity potentials are obtained by the strip approximation. Numerical results are presented in graphs and pertinent discussions are made. It is found that the maximum magnitude of the drift force on SWATH ships would be greater than that of equivalent-displacement monohull ships and the mean heel angle induced by waves could be significant for SWATH ships.

1. Introduction

It is a well-known physical phenomenon that when a floating body is subject to surface waves, the body not only undergoes an oscillatory motion but also its mean position shifts due to the drift forces and moments acting on the body.

The drift forces are often referred to as "second-order mean forces." The second-order mean forces can be normally determined by utilizing the first-order velocity potential solution for restrained and unrestrained bodies. There have been numerous investigations conducted on the second-order mean forces such as added resistance in waves, drift force and moment, and suction force and moment for submerged bodies. These investigations have been applied to monohull ships, and a few familiar ones are given in References [1-6].

In this paper an analysis of the second-order mean forces and moments in the horizontal plane is carried out for twin-hull ships at zero forward speed. The analysis for the mean surge and sway forces and yaw moment is an extension of the work of Newman [2],



and the analysis for the steady roll moment is a new derivation. Both analyses are based on the principle of momentum conservation of the fluid surrounding a body.

For monohull ships the steady mean heel moment induced by waves is considered to be insignificant compared to the steady yaw moment, since there exists a restoring mechanism for the roll while there is no such mechanism for the yaw. The foregoing reasons could have been the cause for the absence of the derivation for the steady mean roll moment. However, for twin-hulls such as a semi-submersible configuration which has a relatively large draft as well as a high vertical center of gravity, it is not obvious if the steady mean roll moment induced by waves could be insignificant. One of the main purposes of this paper is to compute the steady mean roll moment for a SWATH ship and find out if this quantity should be treated as important as the drift force and the steady mean yaw moment.

The solution for the velocity potential function involved is obtained by a strip approximation which is commonly exercised in the prediction of ship motion in waves for monohulls [7] and twin-hulls [8].

Numerical results of a small-waterplane-area, twin-hull (SWATH) ship are presented in figures. The steady mean sway force of a monohull ship is also computed, and the results are compared with those of the SWATH ship.

## 2. Analysis for Mean Forces and Moments

The fluid is assumed incompressible, inviscid, and its motion irrotational. The depth of the fluid is infinite, and a chain of progressive plane waves represents the free surface. All motions of body and fluid are assumed small such that boundary conditions can be linearized and imposed on the undisturbed positions of the body and on the calmwater plane. The body in consideration consists of twin-hulls of slender configuration which are rigidly connected above the water. The slenderness of each hull is such that the longitudinal component of the surface normal is small compared to the transverse components. The body has no forward speed but is free to respond to the incident waves.

Cartesian coordinates  $(x,y,z)$  are taken such that the origin is located on the calmwater plane  $z = 0$  directly above or below the center of gravity of the body at rest, the positive  $x$ -axis is directed toward the bow, and the positive  $z$ -axis is vertically upward

(see Figure 1). The fluid motion is assumed to be harmonic in time with the incident wave frequency.

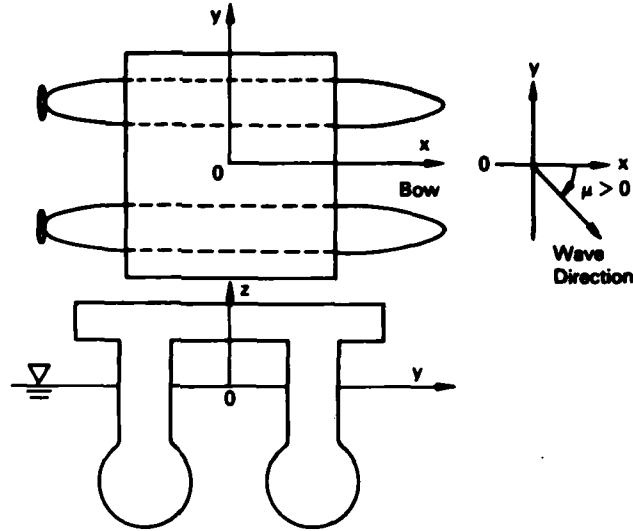


Figure 1 — Description of Coordinate System and Wave-Heading Angle ( $\mu$ )

The fluid velocity  $\underline{V}$  at any point can be represented by the gradient of a velocity potential function  $\phi$  in the form

$$\underline{V} = \nabla\phi(x,y,z,t) = \text{Re}[\nabla\phi(x,y,z)e^{-i\omega t}] \quad (1)$$

where Re means the real part of what follows, and the complex function  $\phi$  is composed of the incident wave potential  $\phi_I$  and the disturbance velocity potential  $\phi_B$ , i.e.,

$$\phi = \phi_I + \phi_B$$

The incident wave potential is given by

$$\phi_I = -\frac{ig\zeta_A}{\omega} \exp(Kz + iKx \cos \mu - iKy \sin \mu) \quad (2)$$

where  $g$  is the gravitational acceleration,  $K = \frac{\omega^2}{g}$ ,  $\zeta_A$  the wave amplitude, and  $\mu$  the angle of wave incidence relative to the positive  $x$ -axis.

The disturbance potential  $\phi_B$  is determined by the solution of the Laplace equation satisfying the following boundary conditions:

$$\phi_{Bz} - K\phi_B = 0^* \quad \text{on } z = 0; \quad (3a)$$

$$\phi_{Bn} = -\phi_{In} + V_n \quad (3b)$$

\*When the spatial variables  $x$ ,  $y$ ,  $z$  and the temporal variable  $t$  are used as a subscript of a function, it means the partial derivative of the function with the respective variable.

on the body surface  $S_0$  at its rest condition, where the subscript  $n$  indicates a normal derivative (the unit normal vector  $\underline{n}$  is positive pointing out of the fluid boundary), and  $V_n$  the normal velocity of the surface;

$$\phi_{Bz} = 0 \quad \text{as } z \rightarrow -\infty \quad (3c)$$

and

$$\lim_{R \rightarrow \infty} \sqrt{R} \left( \frac{\partial \phi_B}{\partial R} - iK\phi_B \right) = 0 \quad (3d)$$

where  $R = \sqrt{(x-\xi)^2 + (y-\eta)^2}$  and  $(\xi, \eta)$  represents a point on  $S_0$ .

The far-field behavior of  $\phi_B$  is well known and is given by Wehausen and Laitone [8] as

$$\phi_B = \left( \frac{K}{2\pi R} \right)^{\frac{1}{2}} H(K, \pi + \theta) \exp \left( Kz + iKR + i \frac{\pi}{4} \right)$$

where  $H(k, \theta)$ , referred to as the Kochin function, is defined by

$$H(k, \theta) = \iint_{S_0} \left[ \phi_{Bn}(\xi, \eta, \zeta) - \phi_B \frac{\partial}{\partial n} \right] \exp(k\zeta + ik\xi \cos \theta + ik\eta \sin \theta) ds \quad (4)$$

From the conservation principle of the rate of change of the linear and angular momentum in the fluid volume  $V$  enclosed by the surfaces  $S_0$ ,  $S_F$  ( $z = 0$  plane excluding the waterplane area  $S_W$ ) and  $S_\infty$  (the far-field control surface), we can obtain the expressions for the mean surge force  $\bar{F}_1$ , sway force  $\bar{F}_2$  and yaw moment  $\bar{F}_6$  (see e.g., Newman [2]\*) and the mean roll moment  $\bar{F}_4$  (see Appendix I) as

$$\begin{aligned} \bar{F}_1 = & - \frac{\rho K^2}{8\pi} \int_0^{2\pi} |H(K, \pi + \theta)|^2 \cos \theta \, d\theta \\ & + \frac{\rho}{2} \omega \zeta_A \cos \mu \operatorname{Im} H(K, \pi - \mu) \end{aligned} \quad (5)$$

$$\begin{aligned} \bar{F}_2 = & - \frac{\rho K^2}{8\pi} \int_0^{2\pi} |H(K, \pi + \theta)|^2 \sin \theta \, d\theta \\ & - \frac{\rho}{2} \omega \zeta_A \sin \mu \operatorname{Re} H(K, \pi - \mu); \end{aligned} \quad (6)$$

$$\begin{aligned} \bar{F}_6 = & \frac{\rho K}{8\pi} \operatorname{Im} \int_0^{2\pi} H^*(K, \theta) \frac{\partial}{\partial \theta} H(K, \theta) d\theta \\ & + \frac{\rho \omega}{2K} \zeta_A \operatorname{Im} \frac{\partial}{\partial \theta} H(K, \pi - \mu) \end{aligned} \quad (7)$$

\*The slight difference in the expressions are due to the difference in the expression of the incident-wave potential.

where the asterisk means the complex conjugate of the function;

$$\bar{F}_1 = \bar{F}_1^{(IB)} + \bar{F}_1^{(BB)} \quad (8)$$

where

$$\begin{aligned} \bar{F}_1^{(IB)} = & \frac{\rho \omega \zeta_A}{2} \operatorname{Re} \iint_{S_0} (z \sin \mu + iy) \left( \phi_{Bn} - \phi_B \frac{\partial}{\partial n} \right) \\ & e^{K(z - ix \cos \mu + iy \sin \mu)} ds, \end{aligned} \quad (8a)$$

and

$$\begin{aligned} \bar{F}_1^{(BB)} = & - \frac{\rho K^2}{8\pi} \operatorname{Re} \int_0^{2\pi} d\theta H^*(K, \theta) \iint_{S_0} (z \sin \theta + iy) \\ & \left( \phi_{Bn} - \phi_B \frac{\partial}{\partial n} \right) e^{K(z + ix \cos \theta + iy \sin \theta)} ds \\ & - \frac{\rho}{16\pi^2} \operatorname{Re} i \int_0^{2\pi} d\theta \int_0^\infty dk \frac{k(k+K)}{k-K} H^* \iint_{S_0} (z \sin \theta + iy) \\ & \left( \phi_{Bn} - \phi_B \frac{\partial}{\partial n} \right) e^{k(z + ix \cos \theta + iy \sin \theta)} ds. \end{aligned} \quad (8b)$$

It should be noted that the expressions for the  $\bar{F}_1$ 's shown above consist of two terms which are  $O(H^2)$  and  $O(H)$ . From Equations (8a) and (8b),  $\bar{F}_1^{(IB)} = O(H)$  and  $\bar{F}_1^{(BB)} = O(H^2)$ . We can readily identify that the terms of  $O(H)$  in the above equations represent the interaction between  $\phi_I$  and  $\phi_B$ , and the terms of  $O(H^2)$  represent the interaction of  $\phi_B$  by itself.

Within the linear analysis the disturbance potential  $\phi_B$  can be expressed as the sum of the diffracted wave potential  $\phi$ , and the motion-related potentials  $\phi_j$  for  $j = 1, 2, \dots, 6$ , i.e.,

$$\phi_B = \sum_{j=1}^7 \xi_j \phi_j \quad (9)$$

where  $\xi_j$  for  $j = 1, 2, \dots, 6$  is the complex amplitude of the  $j$ th mode of motion, i.e.,  $j = 1, 2$  and  $3$  correspond to surge, sway and heave, respectively and  $j = 4, 5$  and  $6$  correspond to roll, pitch and yaw, respectively, and  $\xi_7$  is equal to the incident wave amplitude  $\zeta_A$ .

For the solution of  $\phi_j$  an assumption of two-dimensional flow condition at each cross section is made, and application of the

method of distributing pulsating sources, which satisfy the free-surface condition (3a), on the immersed contours of the cross sections is made [9,10]. In this way, the hydrodynamic interactions between the two cross sections in the same y-z planes are accounted for in the solution of  $\phi_j$ . The motion amplitudes  $\xi_j$  are obtained from coupled linear equations of motion using strip theory [11].

The Kochin function H can be expressed similarly by

$$H = \sum_{j=1}^7 \xi_j H_j \quad (10)$$

where

$$H_j(k, \theta) = \iint_{S_0} \left( \phi_{jn} - \phi_j \frac{\partial}{\partial n} \right) e^{k(\zeta + i\xi \cos \theta + i\eta \sin \theta)} ds \quad (11)$$

### 3. Results and Discussions

A twin-hull ship of semi-submersible configuration, often referred to as small-waterplane area, twin-hull (SWATH), is chosen for a sample calculation. Its particular dimensions are presented in Table 1 together with a monohull ship. The drift force computed for this monohull will be compared later with that of SWATH.

As can be seen from the expressions for the steady mean forces and moments given by Equations (5) through (8), the Kochin function H is directly involved in these formulae. To obtain H the disturbance potential  $\phi_B$  should be known as shown by Equation (4). The potential  $\phi_B$  is further decomposed into seven components which are associated with the six modes of motion and the wave diffraction as indicated by Equation (9). Thus, the necessary computations should begin with obtaining the  $\phi_j$  for  $j = 1, 2, \dots, 7$ . The solutions of  $\phi_j$ 's are utilized to obtain the hydrodynamic coefficients such as the added mass and damping coefficients and wave excited forces and moments. It is then followed by the computation of motion in six-degrees of freedom. The details leading to this stage are described in Lee and Curphey [11], and will not be repeated here.\*

---

\*The extra condition imposed on  $\phi_B$ , i.e.,  $\phi_B = 0$  on  $S_W$  (see Appendix I) is carried out in the numerical evaluation of  $\phi_B$  in this work. This condition is also used to remove the so-called irregular frequencies pointed out by John [12].

**Table 1**  
**Particular Dimensions of SWATH and Monohull in Full Scale**

| Parameter   | SWATH  | Monohull |
|---|--------|----------|
| Displacement, long tons in s.w.                         | 2802.0 | 1761.0   |
| Length at the Waterline, m                              | 52.5   | —        |
| Length of Main Hull, m                                  | 73.2   | 73.2     |
| Beam of Each Hull at the Waterline, m                   | 2.2    | 13.4     |
| Hull Spacing Between the Centerlines, m                 | 22.9   | —        |
| Draft at the Midship, m                                 | 8.1    | 3.62     |
| Bridging Structure Clearance from Waterline, m          | 6.1    | —        |
| Maximum Diameter of Main Hull, m                        | 4.6    | —        |
| Longitudinal Center of Gravity Aft of Main Hull Nose, m | 36.5   | 38.0     |
| Vertical Center of Gravity above Baseline, m            | 10.4   | 5.3      |
| Transverse $\overline{GM}$ , m                          | 2.9    | 1.91     |
| Longitudinal $\overline{GM}$ , m                        | 6.8    | —        |
| Radius of Gyration for Pitch                            | 16.9   | 18.3     |
| Radius of Gyration for Roll                             | 10.2   | 5.3      |
| Waterplane Area, m <sup>2</sup>                         | 193.9  | —        |
| Maximum Sectional Area, Demihull, m <sup>2</sup>        | —      | 40.8     |
| Projected Side Area, Demihull, m <sup>2</sup>           | —      | —        |
| Neutral Axis Height from Waterline, m                   | —      | —        |
| Heave Period, sec                                       | 9.2    | —        |
| Roll Period, sec  | 18.7   | 6.35     |
| Model Scale Ratio                                       | 1/22.5 | 1/11.682 |

Once  $\phi_j$  and  $\xi_j$  are known,  $\phi_B$  can be obtained by Equation (9) and H by Equation (4). The motion amplitudes  $|\xi_j|$  which are normalized by appropriate factors are shown in Figure 2 for beam ( $\mu = 90^\circ$ ), bow-quartering ( $\mu = 135^\circ$ ), and head ( $\mu = 180^\circ$ ) waves. The ship length L used in the figures and text is the length of the submerged main hull, and the notation S used in the normalization of the roll amplitudes means one-half of the separation distance between the longitudinal centerplanes of individual hulls. It can be noted in Figure 2 that the non-dimensionalized peak heave amplitude occurs at the wave length ( $\lambda$ ) which is about twice the ship length. The corresponding peak values for monohulls of equivalent displacement would occur at about  $\lambda/L = 1.0$  to  $1.5$ . The reason for the SWATH having its peak value at longer wave lengths compared to monohulls is due to its longer natural heave period owing to its small waterplane area. The large sway motion in beam waves can be observed at about wave length being equal to 0.4 times the ship length which corresponds to about 1.4 times

the separation distance between the twin struts. From the intermediate results, it was found that in the vicinity of  $\lambda = 0.4L$  the sway added mass becomes negative. This is a particular feature associated with twin-hull ships indicating a strong hydrodynamic interaction between the two hulls. The negative sway added mass was confirmed for a semi-submersible twin cylinder by experiment [11]. It can be seen in Figure 3 that the spiked behavior of the sway motion at  $\lambda = 0.4L$  influences the behavior of the steady mean sway force and roll moment in a similar manner.

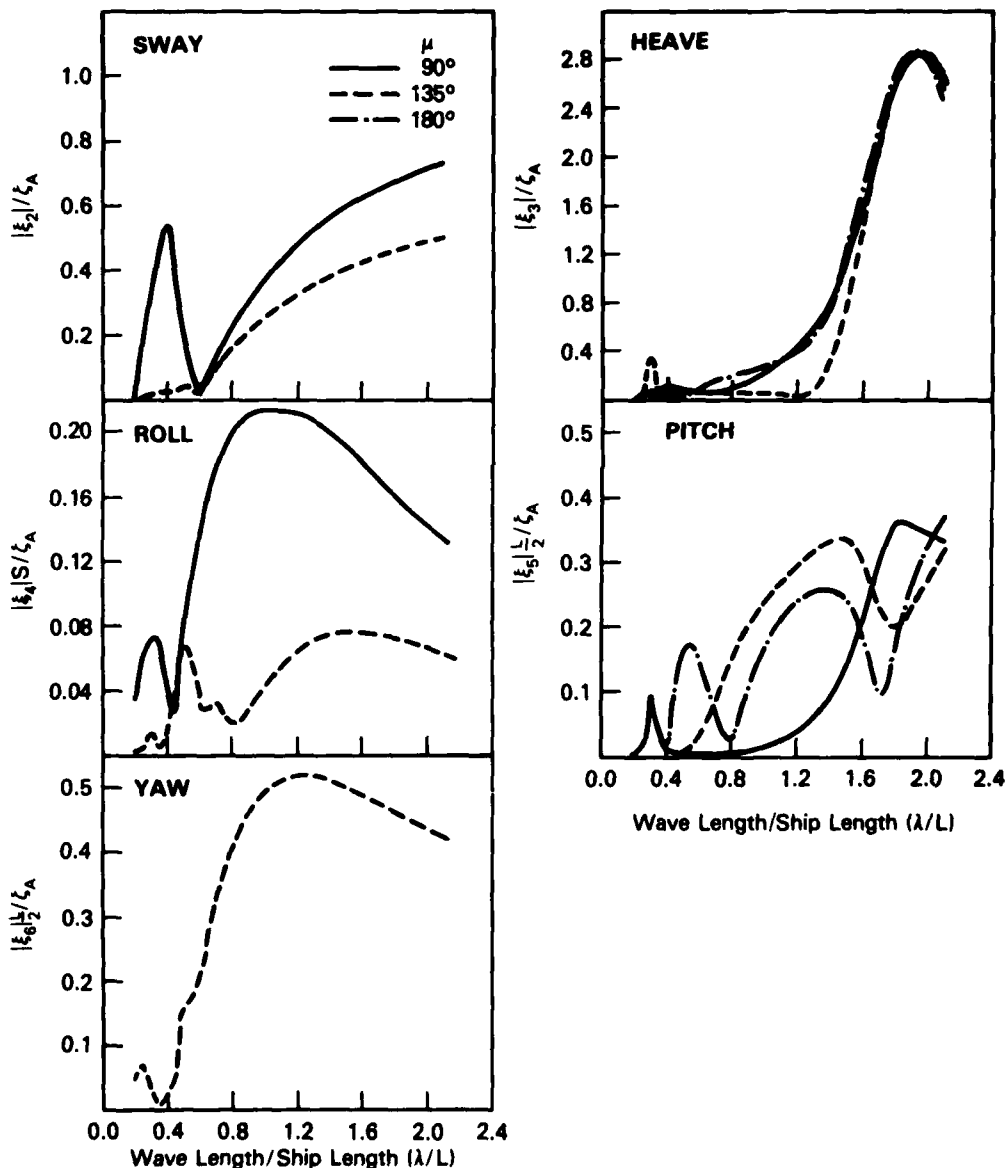


Figure 2 — Motion Amplitude for Beam, Bow-Quartering and Head Waves for SWATH

Steady mean forces and moments on the SWATH subject to beam, bow-quartering and head waves are shown in Figure 3. The forces and moments are normalized by the factors  $\rho g \zeta_A^2 L$  and  $\rho g \zeta_A^2 L^2$ , respectively.

As expected from Equation (5), the steady mean surge force induced by beam waves is negligibly small, since the second term in the right-hand side of the equation which yields a dominant contribution is zero for  $\mu = 90^\circ$ . The first term involving the square of the Kochin function is usually of higher order than the second term because  $H = O(\phi_B)$  for a slender body. One can see in Figure 3 that the magnitude of the first term of Equation (5) contributing to the steady mean surge force is represented by the solid curve ( $\mu = 90^\circ$ ) and is negligibly small. However, when the wave heading is increased to  $135^\circ$  and to  $180^\circ$ , the magnitude of the mean surge force is increased. The direction of the steady mean surge force is in the direction to drift the ship backward. As expected, the steady mean sway force, which will be referred to as drift force hereafter, decreases as the wave heading is changed from the beam waves to the bow-quartering waves. At  $\mu = 180^\circ$ , the drift force should be negligible due to the symmetry of the body.

As already explained earlier, the contributions of the term involving  $|H|^2$  to the steady mean forces and moments are relatively small compared to the term involving the first power of  $H$ . It was found from the computation of  $\bar{F}_1$  and  $\bar{F}_2$  that the square term of  $H$  is less than ten percent of the first-power term of  $H$ . Based on this finding, the computation for  $\bar{F}_3$  is carried out by neglecting  $\bar{F}_3^{(BB)}$  since  $\bar{F}_3^{(BB)}$  as given by Equation (8b) is of  $O(H^2)$ . It was felt that the computation of  $\bar{F}_3^{(BB)}$ , which would require two to three times greater computer time than that required to evaluate  $\bar{F}_3^{(IB)}$ , does not justify its merit. Thus, the results shown in Figure 3 for the steady mean roll moment are represented by  $\bar{F}_3^{(IB)}$  only. The change in sign of the steady roll moment for shorter wave lengths is worth noting. The positive roll moment is the moment to heel the ship toward the lee side of the wave incidence.

The steady mean yaw moment is also shown in Figure 3. For beam waves, the steady mean yaw moment seems insignificant. The mean yaw moment for the bow-quartering waves is found to be the same order of magnitude as that of a monohull ship of equivalent displacement [2].



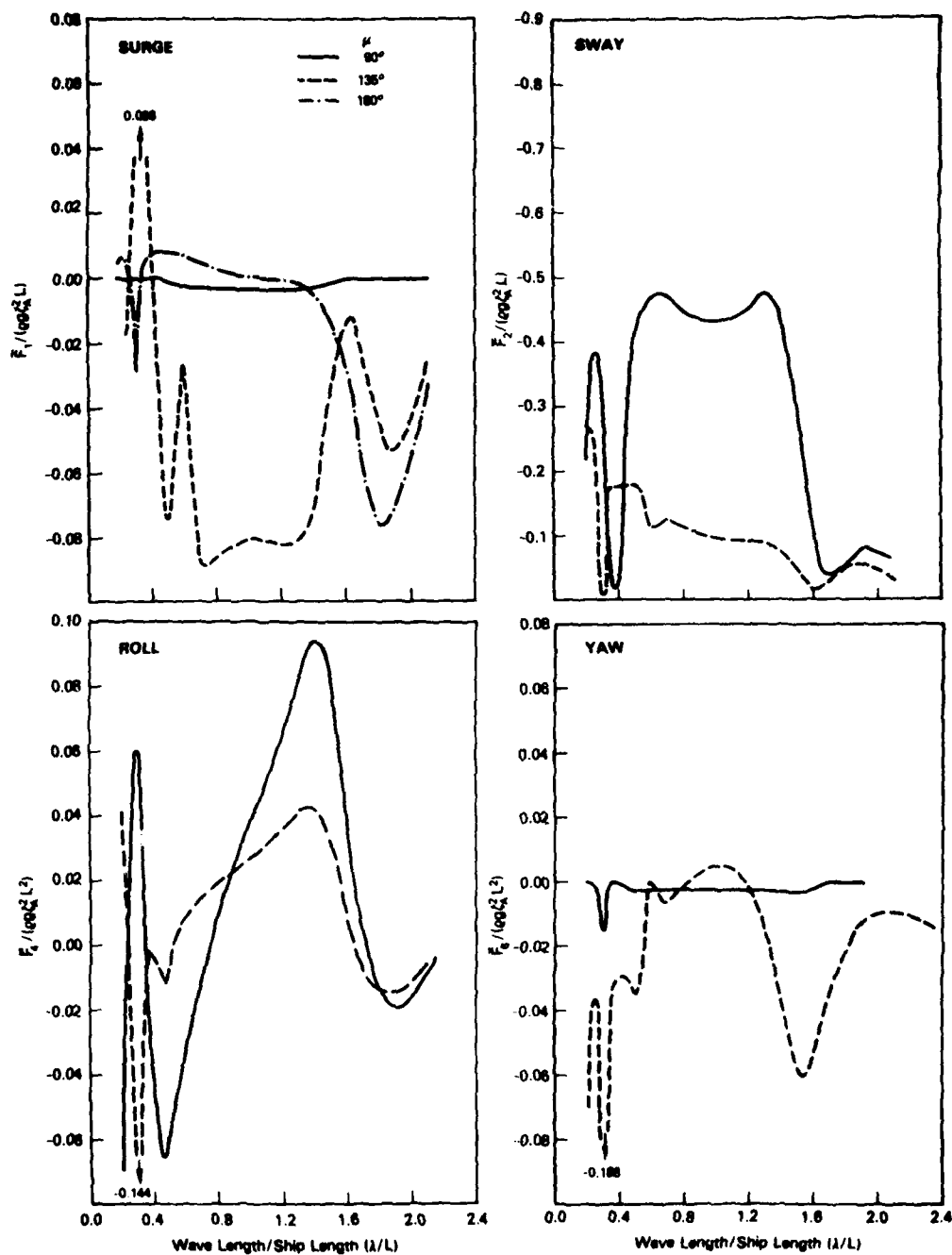


Figure 3 – Steady Mean Forces and Moments for Beam, Bow-Quartering and Head Waves for SWATH

At about  $\lambda/L = 0.3$  the steady mean forces and moments shown in Figure 3 exhibit a singular behavior. The wave length corresponding to about  $0.3L$  happens to be equal to the separation distance between the inner sides of the vertical struts of the SWATH. It is mathematically well-known that there exists a breakdown of the velocity-potential solution at the wave lengths which are integer multiples of the separation between two vertical barriers. This phenomenon was numerically and experimentally confirmed by Lee, et al. [9] for infinitely long horizontal twin cylinders. In the present computation, the strip approximation based on the two-dimensional solution of the velocity potential function has been utilized. Thus, the large spiked behaviors of the steady forces at this wave length are regarded to be a direct consequence of the aforementioned shortcoming of the strip approximation. Although the phenomenon could have been exaggerated by the two-dimensional approximation, the existence of such a phenomenon for a SWATH having long surface-piercing struts cannot be ruled out entirely. At DTNSRDC peculiar phenomena have often been observed in the roll behavior of SWATH models in waves. These phenomena are the asymmetric roll motion and the resonant-like motion at the wave lengths far shorter than the expected resonant wave length. These phenomena could be related to the rapidly oscillating behaviors shown in Figure 3 for  $\lambda/L < 0.6$ .

To investigate the cause for the oscillatory behaviors of the curves in Figure 3 for  $\lambda/L < 0.6$ , computations are made by setting the motion of the body to be zero, i.e.,  $|\xi_j| = 0$  for  $j = 1, 2, \dots, 6$ . In this way, the effect of the motion on the steady mean forces and moments could be investigated. Figure 4 shows the steady mean sway force and roll moment for restrained and unrestrained conditions for beam waves. One can observe that the large hump and hollow for the unrestrained condition diminishes for the restrained condition for  $\lambda/L < 0.6$ . This implies that for  $\lambda/L < 0.6$  the motion-associated hydrodynamic effects have greater influence on the steady forces and moments than the wave diffraction effect does. That is, the disturbance potentials  $\phi_j$  for  $j = 1, 2, \dots, 6$  have greater influence than  $\phi$ , in shorter wave lengths. This implication can be related to the hump-and-hollow behavior of sway and roll motion for  $\lambda/L < 0.6$  shown in Figure 2. It is plausible that the hydrodynamic interactions between the two hulls which are caused by the sway and roll motion could have a direct effect on the oscillatory behavior in the steady forces and moments in shorter wave

lengths. However, it can be realized readily that for  $0.6 < \lambda/L < 1.6$  the trends and the magnitudes of the two curves in Figure 4 are not so different. This implies that the wave diffraction effect which entirely governs the steady mean forces and moments on a restrained body seems the dominant factor in this wave-length range. As wave length increases, the diffraction effect should diminish, and it seems from Figure 4 that such a trend holds true.

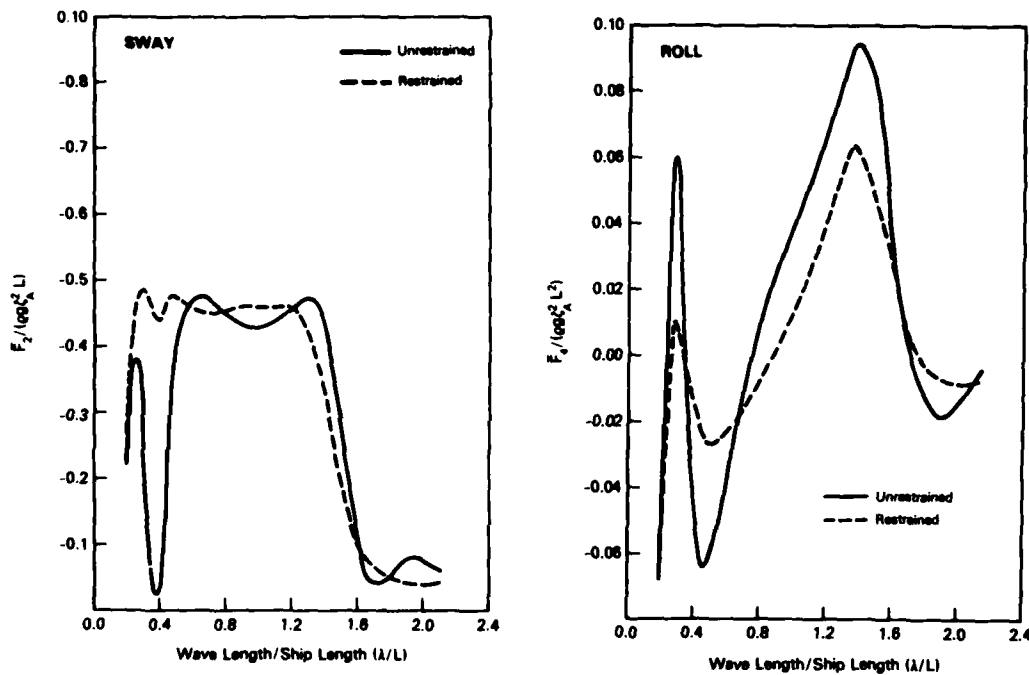


Figure 4 - Steady Mean Sway Force and Roll Moment for Restrained and Unrestrained Condition for SWATH

To assess the maximum magnitudes of the steady mean forces and moments, a wave length-to-amplitude ratio of 60 is assumed, and the calculated values are shown in Table 2 in which the forces and moments are respectively normalized by the displacement of the ship and the displacement times one-quarter length of the ship. The values in Table 2 are obtained by taking the values from Figure 3.

**Table 2**  
**Maximum Steady Mean Forces and Moments for  $\lambda/\zeta_A = 60$  for SWATH**

|             | $ F_1 /\Delta$ | $ F_2 /\Delta$ | $ F_4 /(\Delta \cdot \frac{1}{2})$ | $ F_6 /(\Delta \cdot \frac{1}{2})$ |
|-------------|----------------|----------------|------------------------------------|------------------------------------|
|             | 0.75%          | 3.38%          | 2.92%                              | 2.17%                              |
| $\lambda/L$ | 1.9            | 1.4            | 1.4                                | 1.6                                |
| $\beta$     | 135°           | 90°            | 90°                                | 135°                               |

The heeling angle induced by the steady mean roll moment given in Table 2 can be obtained by

$$\theta = \frac{\bar{F}_r}{\Delta \overline{GM}} \times \frac{180}{\pi} = 10.6^\circ$$

where  $\Delta = 2802$  long tons and  $\overline{GM} = 2.9$  m (see Table 1). The heeling angle is toward the lee side of the wave incidence. Such a steady heeling moment induced by the body and wave motions may explain the observed asymmetric roll motion of SWATH models in waves. That is, when a SWATH model is subject to regular beam waves at zero speed, the oscillatory roll motion often takes place about the vertical plane which is somewhat inclined toward the lee side of the wave incidence.

In Figure 5 the steady mean sway forces of SWATH and a monohull having the particular dimensions as shown in Table 1 are compared. Both ships have the same length. To compare the drift force in a meaningful way, the drift forces are normalized by the factor  $\Delta(\zeta_A/L)^2$  and are shown versus the ratio of wave length to draft (T) of individual ships. It can be observed that at about  $\lambda/T = 6$  two curves intersect and for the smaller values of  $\lambda/T$  the monohull curve shows greater values than the SWATH curve, and the trend reverses for the greater values of  $\lambda/T$ . Since the drift force is proportional to the square of the wave height, and the wave heights for longer waves would be larger than for shorter wave lengths, it is anticipated that SWATH ships can be subject to larger drift forces than monohull ships. This would mean that if a SWATH type of configuration is considered for a drilling ship, a more powerful position control device than for an equivalent monohull ship may be necessary.

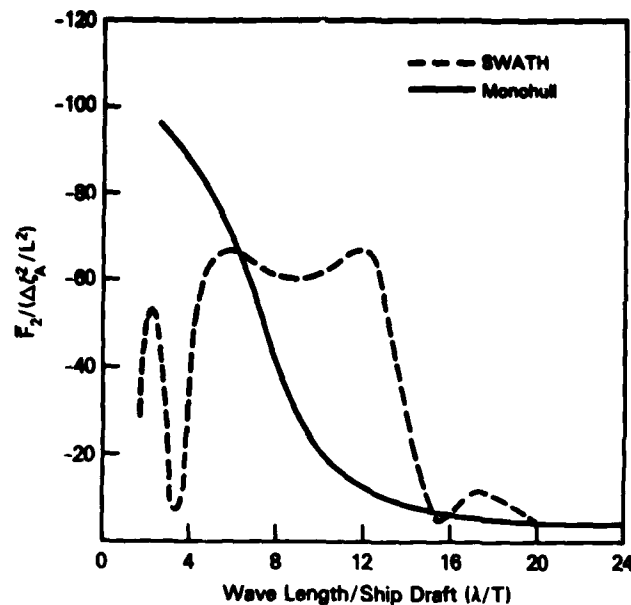


Figure 5 - Steady Mean Sway Force of SWATH and Monohull Ship in Beam Waves

#### 4. Conclusion

Within the limit of the present investigation it is found that SWATH ships can be subject to a larger drift force than equivalent-displacement monohull ships for wave lengths greater than six times the draft of the SWATH. This means that a SWATH may require more power for position control than a monohull of equivalent displacement. The magnitude of the steady heel moment exerted on a SWATH ship could be sufficiently large to cause the ship to assume a heeled mean position resulting in an asymmetric roll oscillation. The results obtained in the present investigation indicate that the wave-diffraction has a significant effect on the steady mean forces and moments on a SWATH ship for  $2.0 < \lambda/(2S) < 5.0$ .

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### Appendix I - Derivation of Mean Roll Moment

The rate of change of angular momentum of the fluid in the volume  $V$  enclosed by the surface  $S$  is given by

$$\begin{aligned}
 \frac{dK}{dt} &= \rho \frac{d}{dt} \iiint_V \underline{r} \times \underline{v} \, dv = \rho \iiint_V \underline{r} \times \frac{\partial \underline{v}}{\partial t} \, dv + \rho \iint_S \underline{r} \times \underline{v} U_n \, ds \\
 &= \rho \iiint_V \underline{r} \times \left\{ -(\underline{v} \cdot \nabla) \underline{v} - \frac{\nabla p}{\rho} - \nabla(gz) \right\} \, dv + \rho \iint_S \underline{r} \times \underline{v} U_n \, ds \\
 &= -\rho \iiint_V \left[ \nabla \cdot \{ \underline{v} (\underline{r} \times \underline{v}) \} + \underline{r} \times \nabla \left( \frac{p}{\rho} + gz \right) \right] \, dv \\
 &\quad + \rho \iint_S \underline{r} \times \underline{v} U_n \, ds \\
 &= -\rho \iint_S \left[ (\underline{r} \times \underline{v})(V_n - U_n) + \frac{p}{\rho} (\underline{r} \times \underline{n}) + gz \underline{r} \times (0, 0, n_z) \right] \, ds \quad (I-1)
 \end{aligned}$$

where  $\underline{v}$  is the velocity vector of the fluid,  $U_n$  is the normal velocity of the surface  $S$ ,  $V_n$  the normal fluid velocity on  $S$ ,  $\underline{r}$  the position vector and  $n_z$  the z-component of the unit normal vector  $\underline{n}$  on  $S$ .

If we let  $S = S_b + S_F + S_\infty$  where  $S_b$  represents the body surface,  $S_F$  the  $z = 0$  plane excluding the waterplane surface of the body  $S_W$  and  $S_\infty$  the side and the bottom surface of a vertical cylinder having a large radius  $R$ , we can show from Equation (I-1) that the moment vector  $\underline{M}$  about the origin is given by

$$\begin{aligned}
 \underline{M} &\equiv \iint_{S_b} p \underline{r} \times \underline{n} \, ds \\
 &= - \frac{dK}{dt} - \rho g \iint_{S_b} z(yn_z, -xn_z, 0) \, ds \\
 &\quad - \rho \iint_{S_F} (x, y, 0) \times \nabla \phi_z \, dx \, dy - \iint_{S_F} (x, y, 0) \times \underline{n} p \, dx \, dy \\
 &\quad - \rho \iint_{S_R} p(\underline{r} \times \underline{n}) \, ds - \rho \iint_{S_R} (\underline{r} \times \nabla \phi) \phi_n \, ds \quad (I-2)
 \end{aligned}$$

where  $\nabla \phi = \underline{v}$ , and  $S_R$  is the side surface of the vertical cylinder.

The x-component of the time-average of Equation (I-2) is obtained as

$$\begin{aligned} \bar{F}_x = & -\rho \iint_{S_F} \left( \phi_z^2 - \frac{1}{2} |\nabla \phi|^2 \right) y \, dx \, dy \\ & -\rho \iint_{S_R} \left\{ \frac{z}{2} |\nabla \phi|^2 n_z + (y \phi_z - z \phi_y) \phi_n \right\} ds \end{aligned} \quad (I-3)$$

since  $dK/dt$  is zero due to the conservation principle,  $\iint_{S_0} y z n_z \, ds = 0$  due to the symmetry of the body within  $O(\phi^2)$ , and  $\iint_{S_F} \bar{\phi}_t y \, dx \, dy = \iint_{S_R} \bar{\phi}_t z \, ds = 0 + O(\phi^2)$ .

Using the cylindrical coordinate system,  $x = R \cos \theta$  and  $y = R \sin \theta$ , we can rewrite Equation (I-3) as

$$\begin{aligned} \bar{F}_x = & \frac{\rho}{2} \iint_{S_F} y (\phi_x^2 + \phi_y^2 - \phi_z^2) \, dx \, dy \\ & -\rho R \int_0^{2\pi} d\theta \int_{-\infty}^0 \left\{ \frac{z}{2} n_z |\nabla \phi|^2 + (y \phi_z - z \phi_y) \phi_n \right\} dz \end{aligned}$$

Since the asymptotic behavior of  $\phi$  as  $R \rightarrow \infty$  can be expressed as  $\phi \sim e^{Kz} \phi(x, y, 0, t)$ , we can show that

$$\begin{aligned} \bar{F}_x = & \frac{\rho}{2} \iint_{S_F} y (\phi_x^2 + \phi_y^2 - \phi_z^2) \, dx \, dy \\ & + \frac{\rho}{8K^2} \int_{C_R} n_z (\phi_x^2 + \phi_y^2 + K^2 \phi^2) \, dl \\ & -\rho \int_{C_R} \left( \frac{y}{2} \phi + \frac{1}{4K^2} \phi_y \right) \phi_n \, dl \end{aligned} \quad (I-4)$$

where  $\int_{C_R} dl = \int_0^{2\pi} R \, d\theta$  and the elementary integrals such as  $\int_{-\infty}^0 e^{2Kz} dz = \frac{1}{2K}$  and  $\int_{-\infty}^0 z e^{2Kz} dz = \frac{1}{4K^2}$  were used.

From the Gauss theorem, we have

$$\begin{aligned} \int_{C_R} n_z (\phi_x^2 + \phi_y^2 + K^2 \phi^2) \, dl = & 2 \iint_{S_F} (\phi_x \phi_{xy} + \phi_y \phi_{yy} + K^2 \phi \phi_y) \, dx \, dy \\ & + \int_{C_W} n_z (\phi_x^2 + \phi_y^2 + K^2 \phi^2) \, dl, \end{aligned} \quad (I-5)$$

and

$$\int_{C_R} \left( \frac{y}{2} \phi + \frac{1}{4K^2} \phi_y \right) \phi_n \, d\ell = \frac{1}{2} \iint_{S_F} \left\{ \left( \phi \phi_y + y |\tilde{v} \phi|^2 + y \phi \tilde{v}^2 \phi \right) + \frac{1}{2K^2} \left( \tilde{v} \phi_y \cdot \tilde{v} \phi + \phi_y \tilde{v}^2 \phi \right) \right\} ds + \int_{C_W} \left( \frac{y}{2} \phi + \frac{1}{4K^2} \phi_y \right) \phi_n \, d\ell \quad (I-6)$$

where  $\int_{C_W} d\ell$  is the integral along the contours of the waterplane area of the body in the counterclockwise direction, and  $\tilde{v} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right)$ .

Substitution of Equations (I-5) and (I-6) into Equation (I-4) yields

$$\begin{aligned} \bar{F}_x = & -\frac{\rho}{2} \iint_{S_F} \left( \phi_{xx} + \phi_{yy} + K^2 \phi \right) \left( \frac{1}{2K^2} \phi_y + y \phi \right) dx dy \\ & + \frac{\rho}{8K^2} \int_{C_W} n_2 \left( \phi_x^2 + \phi_y^2 + K^2 \phi^2 \right) d\ell \\ & - \rho \int_{C_W} \left( \frac{y}{2} \phi + \frac{1}{4K^2} \phi_y \right) \phi_n \, d\ell \end{aligned} \quad (I-7)$$

We arbitrarily assume that the velocity potential  $\phi$  can be analytically extended onto the waterplane area  $S_W$ . Then, we can write

$$\begin{aligned} \bar{F}_x = & -\frac{\rho}{2} \iint_{S_F + S_W} \left( \phi_{xx} + \phi_{yy} + K^2 \phi \right) \left( \frac{1}{2K^2} \phi_y + y \phi \right) dx dy \\ & + \frac{\rho}{2} \iint_{S_W} \left( \phi_{xx} + \phi_{yy} + K^2 \phi \right) \left( \frac{1}{2K^2} \phi_y + y \phi \right) dx dy \\ & + \frac{\rho}{8K^2} \int_{C_W} n_2 \left( \phi_x^2 + \phi_y^2 + K^2 \phi^2 \right) d\ell \\ & + \rho \int_{C_W} \left( \frac{y}{2} \phi + \frac{1}{4K^2} \phi_y \right) \phi_n \, d\ell \end{aligned} \quad (I-8)$$

If we apply the same procedure of deriving Equation (I-7) from (I-4) on the domain  $S_W$ , we can show that



$$\begin{aligned}
& \frac{\rho}{2} \iint_{S_W} (\phi_{xx} + \phi_{yy} + K^2 \phi) \left( \frac{1}{2K^2} \phi_y + y\phi \right) dx dy \\
&= \frac{\rho}{2} \iint_{S_W} y (\phi_x^2 + \phi_y^2 - \phi_z^2) dx dy \\
&- \frac{\rho}{8K^2} \int_{C_W} n_z (\phi_x^2 + \phi_y^2 + K^2 \phi^2) dl \\
&- \rho \int_{C_W} \left( \frac{y}{2} \phi + \frac{1}{4K^2} \phi_y \right) \phi_n dl
\end{aligned}$$

Substituting the foregoing identity into Equation (I-8), we can derive

$$\begin{aligned}
\bar{F}_s &= - \frac{\rho}{2} \iint_{S_F + S_W} (\phi_{xx} + \phi_{yy} + K^2 \phi) \left( \frac{1}{2K^2} \phi_y + y\phi \right) dx dy \\
&+ \frac{\rho}{2} \iint_{S_W} y (\phi_x^2 + \phi_y^2 - \phi_z^2) dx dy \\
&= - \frac{\rho}{8K^2} \iint_{S_F + S_W} (\phi_y^* + 2K^2 y \phi^*) (\tilde{v}^2 + K^2) \phi dx dy \\
&+ \frac{\rho}{4} \iint_{S_W} y (\phi_x \phi_x^* + \phi_y \phi_y^* - \phi_z \phi_z^*) dx dy \tag{I-9}
\end{aligned}$$

We impose the atmospheric pressure condition for  $\phi_B$  on  $S_W$ , i.e.,  $\phi_B = 0$  on  $S_W$  and obtain

$$\begin{aligned}
\bar{F}_s &= - \frac{\rho}{8K^2} \iint_{S_F + S_W} (\phi_y^* + 2K^2 y \phi^*) (\tilde{v}^2 + K^2) \phi dx dy \\
&- \frac{\rho}{4} \iint_{S_W} y \{ (\phi_{Iz} + \phi_{Bz}) (\phi_{Iz}^* + \phi_{Bz}^*) \\
&- (\phi_{Ix} \phi_{Ix}^* + \phi_{Iy} \phi_{Iy}^*) \} dx dy, \tag{I-10}
\end{aligned}$$

since  $\phi_B = 0$  on  $S_W$  implies that  $\phi_{Bx} = \phi_{By} = 0$  on  $S_W$ .

We can show that

$$\begin{aligned}
& \iint_{S_W} y(\phi_{Ix}\phi_{Ix}^* + \phi_{Iy}\phi_{Iy}^* - \phi_{Iz}\phi_{Iz}^*) dx dy \\
& = \iint_{S_W} y(-K^2 - K^2) \left(\frac{g\zeta_A}{\omega}\right)^2 dx dy \\
& = -2gK\zeta_A^2 \iint_{S_W} y dx dy = 0
\end{aligned}$$

due to the symmetry of the waterplane area of the twin hulls. Thus, Equation (I-10) reduces to

$$\begin{aligned}
\bar{F}_* = & -\frac{\rho}{8K^2} \iint_{S_F+S_W} (\phi_y^* + 2K^2y\phi^*)(\tilde{\nabla}^2 + K^2)\phi dx dy \\
& - \frac{\rho}{4} \iint_{S_W} y|\phi_{Bz}|^2 dx dy - \frac{\rho}{2} \operatorname{Re} \iint_{S_W} y \phi_{Iz}\phi_{Bz}^* dx dy \quad (I-11)
\end{aligned}$$

If we assume that  $\phi_{Bz}$  and  $\phi_{Iz}$  are approximately only functions of  $x$  on each waterplane of the two struts, we can neglect the last two integrals in Equation (I-11). Furthermore, since  $(\tilde{\nabla}^2 + K^2)\phi_I = 0$ , we finally obtain

$$\bar{F}_* = -\frac{\rho}{8K^2} \iint_{S_F+S_W} \{(\phi_{Iy} + \phi_{By})^* + 2K^2y(\phi_I^* + \phi_B^*)\}(\tilde{\nabla}^2 + K^2)\phi_B dx dy \quad (I-12)$$

The foregoing expression can be divided into two parts according to the products of  $\phi_I$  and  $\phi_B$  in the integrand. That is,

$$\bar{F}_*^{(IB)} = -\frac{\rho}{8K^2} \iint_{S_F} (\phi_{Iy}^* + 2K^2y\phi_I^*)(\tilde{\nabla}^2 + K^2)\phi_B dx dy \quad (I-13)$$

$$\bar{F}_*^{(BB)} = -\frac{\rho}{8K^2} \iint_{\bar{S}_F} (\phi_{By}^* + 2K^2y\phi_B^*)(\tilde{\nabla}^2 + K^2)\phi_B dx dy \quad (I-14)$$

where  $\bar{S}_F = S_F + S_W$ .

The expression for  $\phi_B$  derived from Green's theorem can be given by (see, e.g., Equation (74) of [5])

$$\phi_B(x, y, 0) = \frac{1}{4\pi^2} \int_0^{2\pi} d\theta \int_0^\infty dk \frac{k}{k-K} e^{-ik(x \cos \theta + y \sin \theta)} H(k, \theta) \quad (I-15)$$

where the symbol  $\int_0^\infty$  means that the path of the integral is taken below the pole  $k=K$ .

Substituting Equation (I-15) into Equation (I-13) and using Equation (2) for the expression of  $\phi_I$ , we can show that

$$\begin{aligned}
 \bar{F}_A^{(IB)} &= - \frac{\rho g \zeta_A}{32\pi^2 K^2 \omega} \iint_{\bar{S}_F} (-K \sin \mu + i2K^2 y) e^{-iK(x \cos \mu - y \sin \mu)} \\
 &\quad \int_0^{2\pi} d\theta \int_0^\infty dk \frac{k(K^2 - k^2)}{k - K} H(k, \theta) e^{-ik(x \cos \theta + y \sin \theta)} dx dy \\
 &= \frac{\rho g \zeta_A}{16\pi^2 \omega} \int_0^{2\pi} d\theta \int_0^\infty dk k(k + K) H(k, \theta) \iint_{\bar{S}_F} \left( -\frac{\sin \mu}{2K} + iy \right) \\
 &\quad e^{-ix(K \cos \mu + k \cos \theta) - iy(-K \sin \mu + k \sin \theta)} dx dy \quad (I-16)
 \end{aligned}$$

Similarly, we can derive that

$$\begin{aligned}
 \bar{F}_A^{(BB)} &= - \frac{\rho}{8K^2} \iint_{\bar{S}_F} \left( \frac{\partial}{\partial y} + 2K^2 y \right) \left[ \frac{1}{4\pi^2} \int_0^{2\pi} d\theta \int_0^\infty dk \frac{k}{k - K} \right. \\
 &\quad \left. e^{ik(x \cos \theta + y \sin \theta)} H^*(k, \theta) \right] \left[ \frac{1}{4\pi^2} \int_0^{2\pi} d\theta' \int_0^\infty dk' \frac{k'(K^2 - k'^2)}{k' - K} \right. \\
 &\quad \left. e^{-ik'(x \cos \theta' + y \sin \theta')} H(k', \theta') \right] dx dy \\
 &= \frac{\rho}{128\pi^4 K^2} \int_0^{2\pi} d\theta \int_0^\infty dk \frac{k}{k - K} H^*(k, \theta) \int_0^{2\pi} d\theta' \int_0^\infty dk' k'(k' + K) \\
 &\quad H(k', \theta') \iint_{\bar{S}_F} (ik \sin \theta + 2K^2 y) \\
 &\quad e^{ix(k \cos \theta - k' \cos \theta') + iy(k \sin \theta - k' \sin \theta')} dx dy. \quad (I-17)
 \end{aligned}$$

Use the following relations in Equations (I-16) and (I-17)

$$\int_{-\infty}^{\infty} e^{-iax} da = 2\pi \delta(x),$$

$$\int_{-\infty}^{\infty} a e^{-iax} da = i2\pi \frac{d}{dx} \delta(x) \equiv i2\pi \delta'(x),$$

$$\int_{-\infty}^{\infty} f(\xi) \delta'(x - \xi) d\xi = -f'(x),$$

$$\delta(l - l') \delta(m - m') = \frac{1}{k'} \delta(k - k') \delta(\theta - \theta'),$$

$$\frac{\partial}{\partial m'} = \sin \theta' \frac{\partial}{\partial k'} + \frac{\cos \theta'}{k'} \frac{\partial}{\partial \theta'},$$

where  $\delta$  is the Dirac delta function and

$$\begin{aligned} l &= k \cos \theta, & m &= k \sin \theta \\ l' &= k' \cos \theta', & m' &= k' \sin \theta \end{aligned}$$

Then, we can derive that

$$\begin{aligned} \bar{F}_s^{(IB)} &= \frac{\rho g \zeta_A}{4\omega} \left[ -\sin \mu H(K, \pi - \mu) \right. \\ &\quad \left. + \left( \sin \theta \frac{\partial}{\partial k} + \frac{\cos \theta}{k} \frac{\partial}{\partial \theta} \right) (K+k) H(k, \theta) \right]_{\substack{k=K \\ \theta=\pi-\mu}} \\ &= \frac{\rho \omega \zeta_A}{2} \operatorname{Re} \iint_{S_0} (z \sin \mu + iy) \left( \phi_{Bn} - \phi_B \frac{\partial}{\partial n} \right) \\ &\quad e^{K(z - ix \cos \mu + iy \sin \mu)} ds \end{aligned} \quad (I-18)$$

$$\begin{aligned} \bar{F}_s^{(BB)} &= \frac{i\rho}{32\pi^2 K^2} \int_0^{2\pi} d\theta \sin \theta \int_0^\infty dk \frac{k^2(k+K)}{k-K} |H(k, \theta)|^2 \\ &\quad - \frac{i\rho}{32\pi^2 K^2} \int_0^{2\pi} d\theta \int_0^\infty dk \frac{2kK^2}{k-K} H^* \left( \sin \theta \frac{\partial}{\partial k} + \frac{\cos \theta}{k} \frac{\partial}{\partial \theta} \right) (k+K) H(k, \theta) \\ &= \frac{i\rho}{32\pi^2 K^2} \int_0^{2\pi} d\theta \sin \theta \int_0^\infty dk \frac{k^2(k+K)}{k-K} |H(k, \theta)|^2 \\ &\quad - \frac{i\rho}{16\pi^2} \int_0^{2\pi} d\theta \sin \theta \int_0^\infty dk \frac{k}{k-K} |H|^2 \\ &\quad - \frac{\rho K^2}{8\pi} \int_0^{2\pi} d\theta H^*(K, \theta) \iint_{S_0} (z \sin \theta + iy) \left( \phi_{Bn} - \phi_B \frac{\partial}{\partial n} \right) \\ &\quad e^{K(z + ix \cos \theta + iy \sin \theta)} ds \\ &\quad - \frac{i\rho}{16\pi^2} \int_0^{2\pi} d\theta \int_0^\infty dk \frac{k(k+K)}{k-K} H^* \iint_{S_0} (z \sin \theta + iy) \left( \phi_{Bn} - \phi_B \frac{\partial}{\partial n} \right) \\ &\quad e^{k(z + ix \cos \theta + iy \sin \theta)} ds \end{aligned}$$

The first two terms of the foregoing expression involved with  $|H|^2$  are pure imaginary values; hence, they do not contribute to  $\bar{F}_s^{(BB)}$  which should be a real function. Thus, we have

$$\begin{aligned}
\bar{F}_x(BB) = & -\frac{\rho K^2}{8\pi} \operatorname{Re} \int_0^{2\pi} d\theta H^*(K, \theta) \iint_{S_0} (z \sin \theta + iy) \left( \phi_{Bn} - \phi_B \frac{\partial}{\partial n} \right) \\
& e^{K(z + ix \cos \theta + iy \sin \theta)} ds \\
& - \frac{\rho}{16\pi^2} \operatorname{Re} i \int_0^{2\pi} d\theta \int_0^\infty dk \frac{k(k+K)}{k-K} H^* \iint_{S_0} (z \sin \theta + iy) \\
& \left( \phi_{Bn} - \phi_B \frac{\partial}{\partial n} \right) e^{k(z + ix \cos \theta + iy \sin \theta)} ds \quad (I-19)
\end{aligned}$$

#### References

- 1 MARUO, H., "The Drift of a Body Floating on Waves," Journal of Ship Research, 4, 1960, pp. 1-10.
- 2 NEWMAN, J.N., "The Drift Force and Moment on Ships in Waves," Journal of Ship Research, 11, 1967, pp. 51-60.
- 3 LIN, W.C. and REED, A., "The Second-Order Steady Force and Moment on a Ship Moving in an Oblique Seaway," Proceedings of Eleventh Symposium on Naval Hydrodynamics, London, 1976, pp. 333-345.
- 4 FALTINSEN, O. and LOKEN A., "Drift Forces and Slowly-Varying Horizontal Forces on a Ship in Waves," Proceedings of Symposium on Applied Mathematics, Delft, 1978, pp. 22-41.
- 5 LEE, C.M. and NEWMAN, J.N., "The Vertical Mean Force and Moment of Submerged Bodies Under Waves," Journal of Ship Research, 15, 1971, pp. 231-245.
- 6 KIM, Y.H., "Computation of the Second-Order Steady Forces Acting on a Surface Ship in an Oblique Wave," David Taylor Naval Ship R&D Center Report, DTNSRDC/SPD 0964-01, 1981.
- 7 SALVESEN, N., TUCK, E.O. and FALTINSEN, O., "Ship Motion and Sea Loads," Transactions of Society of Naval Architects and Marine Engineers, 78, 1970, pp. 250-287.
- 8 WEHAUSEN, J.V. and LAITONE, E.V., "Surface Waves," Handbuch der Physik, Springer-Verlag, Berlin, 9, 1960.
- 9 LEE, C.M., BEDEL, J.W. and JONES, H., "Added Mass and Damping Coefficients of Heaving Twin Cylinders in a Free Surface," David Taylor Naval Ship R&D Center Report 3695, 1971.
- 10 PIEN, P.C. and LEE, C.M., "Motion and Resistance of a Low Waterplane-Area Catamaran," Proceedings of Ninth Symposium on Naval Hydrodynamics, Paris, 1972, pp. 463-545 (see Appendix A).
- 11 LEE, C.M. and CURPHEY, R.M., "Prediction of Motion, Stability, and Wave Load of Small Waterplane Area, Twin-Hull Ships," Transactions of Society of Naval Architects and Marine Engineers, 85, 1977, pp. 94-130.
- 12 JOHN, F., "On the Motion of Floating Bodies: II. Simple Harmonic Motions," Communication of Pure and Applied Mathematics, 13, 1950, pp. 45-101.

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